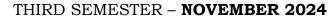
## LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034



Date: 09-11-2024

Time: 01:00 pm-04:00 pm

## M.Sc. DEGREE EXAMINATION – STATISTICS





Max.: 100 Marks

## PST3MC02 - ADVANCED STOCHASTIC PROCESSES

Dept. No.

SECTION A – K1 (CO1)			
	Answer ALL the questions $(5 \times 1 = 5)$		
1	Define the following		
a)	Markov process.		
b)	Pure birth process.		
c)	Renewal function.		
d)	Branching process.		
e)	Brownian motion absorbed at the origin.		
SECTION A – K2 (CO1)			
	Answer ALL the questions $(5 \times 1 = 5)$		
2	Fill in the blanks		
a)	A matrix is called doubly stochastic if		
b)	The Yule process is an example of process.		
c)	The excess life in Poisson process follows distribution.		
d)	Electrical pulses in communication theory are often postulated to describe a process.		
e)	Brownian motion with drift is a stochastic process with the property that every increment $X(t+s)$ –		
	X(s) follows normal distribution with mean		
SECTION B – K3 (CO2)			
	Answer any THREE of the following $(3 \times 10 = 30)$		
3	Explain the following: (i) Types of stochastic process based on index parameter and state space		
	(ii) One-dimensional random walks. (5+5)		
4	Derive the differential equations for a pure birth process.		
5	Narrate Type II counter model with the necessary diagram.		
6	Explain stationary process with the help of two examples.		
7	Write about Brownian motion process.		

SECTION C – K4 (CO3)			
	Answer any TWO of the following	$(2 \times 12.5 = 25)$	
8	(a)Show that one-dimensional random walk is recurrent.		
	(b)Prove that communication is an equivalence relation.	(8.5+4)	
9	Derive P <sub>n</sub> (t) for Yule process and hence find its mean and variance.		
10	State and prove the elementary renewal theorem.		
11	Write in detail about two contrasting stationary processes.		
SECTION D – K5 (CO4)			
	Answer any ONE of the following	$(1 \times 15 = 15)$	
12	Derive forward and backward Kolmogorov differential equations of birth and death pr	ocess.	
13	Establish the generating function relations for branching process and hence find its mea	an and variance.	
SECTION E – K6 (CO5)			
	Answer any ONE of the following	$(1 \times 20 = 20)$	
14	Consider the Markov chain $\{X_n , n \ge 1\}$ with state space $S = \{1,2,3,4,5,6\}$ has the following one-step		
	transition probabilities: $P_{11} = 1/3$ , $P_{13} = 2/3$ , $P_{22} = \frac{1}{2}$ , $P_{23} = \frac{1}{4}$ , $P_{25} = \frac{1}{4}$ , $P_{31} = 2/5$ , $P_{33} = 3/5$ ,		
	$P_{42} = P_{43} = P_{44} = P_{46} = \frac{1}{4}, P_{55} = P_{56} = \frac{1}{2}, P_{65} = \frac{1}{4}, P_{66} = \frac{3}{4}.$		
	(i) Find the equivalence classes		
	(ii) Find periodicity of states		
	(iii) Determine recurrent and transient states.	(3+5+12)	
15	(a) Prove that the variance of a sum as a martingale.		
	(b) Show that Poisson process can be viewed as a renewal process.	(5+15)	

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