LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2024**

PST3MC02 – ADVANCED STOCHASTIC PROCESSES

Date: 09-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)	
	Answer ALL the questions	**(5 x 1 = 5)**
1	**Define the following**	
a)	Markov process.	
b)	Pure birth process.	
c)	Renewal function.	
d)	Branching process.	
e)	Brownian motion absorbed at the origin.	
	SECTION A – K2 (CO1)	
	Answer ALL the questions	**(5 x 1 = 5)**
2	**Fill in the blanks**	
a)	A matrix is called doubly stochastic if -------------------.	
b)	The Yule process is an example of ------------ process.	
c)	The excess life in Poisson process follows -------------- distribution.	
d)	Electrical pulses in communication theory are often postulated to describe a ----------- process.	
e)	Brownian motion with drift is a stochastic process with the property that every increment X(t+s) – X(s) follows normal distribution with mean --------.	
	SECTION B – K3 (CO2)	
	Answer any THREE of the following	**(3 x 10 = 30)**
3	Explain the following: (i) Types of stochastic process based on index parameter and state space (ii) One-dimensional random walks.	(5+5)
4	Derive the differential equations for a pure birth process.	
5	Narrate Type II counter model with the necessary diagram.	
6	Explain stationary process with the help of two examples.	
7	Write about Brownian motion process.	

SECTION C – K4 (CO3)		
	Answer any TWO of the following	**(2 x 12.5 = 25)**
8	(a)Show that one-dimensional random walk is recurrent. (b)Prove that communication is an equivalence relation.	(8.5+4)
9	Derive $P_n(t)$ for Yule process and hence find its mean and variance.	
10	State and prove the elementary renewal theorem.	
11	Write in detail about two contrasting stationary processes.	
SECTION D – K5 (CO4)		
	Answer any ONE of the following	**(1 x 15 = 15)**
12	Derive forward and backward Kolmogorov differential equations of birth and death process.	
13	Establish the generating function relations for branching process and hence find its mean and variance.	
SECTION E – K6 (CO5)		
	Answer any ONE of the following	**(1 x 20 = 20)**
14	Consider the Markov chain $\{X_n , n \geq 1\}$ with state space S = {1,2,3,4,5,6} has the following one-step transition probabilities: P_{11} =1/3 , P_{13} = 2/3 , P_{22} = ½ , P_{23} = ¼ , P_{25} = ¼ , P_{31} = 2/5 , P_{33} =3/5 , P_{42} = P_{43} = P_{44} = P_{46} = ¼ , P_{55} = P_{56} = ½ , P_{65} = ¼ , P_{66} = ¾. (i) Find the equivalence classes (ii) Find periodicity of states (iii) Determine recurrent and transient states.	(3+5+12)
15	(a) Prove that the variance of a sum as a martingale. (b) Show that Poisson process can be viewed as a renewal process.	(5+15)

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